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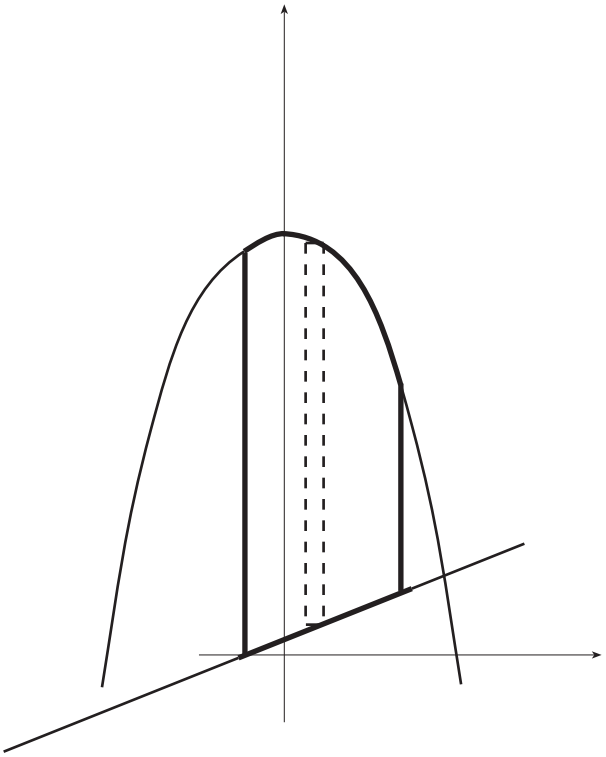
Test 1
Spring 2007
MATH 122 Section 01
February 2, 2002

Directions : You have 50 minutes to complete all 6 problems on this exam. There are a possible 100 points to be earned. You may not use your book, notes, or any graphing/programmable calculator. Please be sure to show all pertinent work. *An incorrect answer with no work will receive no credit!* If any portion of the exam is unclear please come to me and I will elaborate provided I can do so without giving away the problem.

1. (20 points)
Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

$$y = x + 1, \qquad y = 9 - x^2, \qquad x = -1, \qquad x = 2$$

Solution : It seems to me that the best choice is to integrate with repect to x, since the rectangle approximations always start on the curve $y = x+1$ and end on the curve $y = 9 - x^2$.

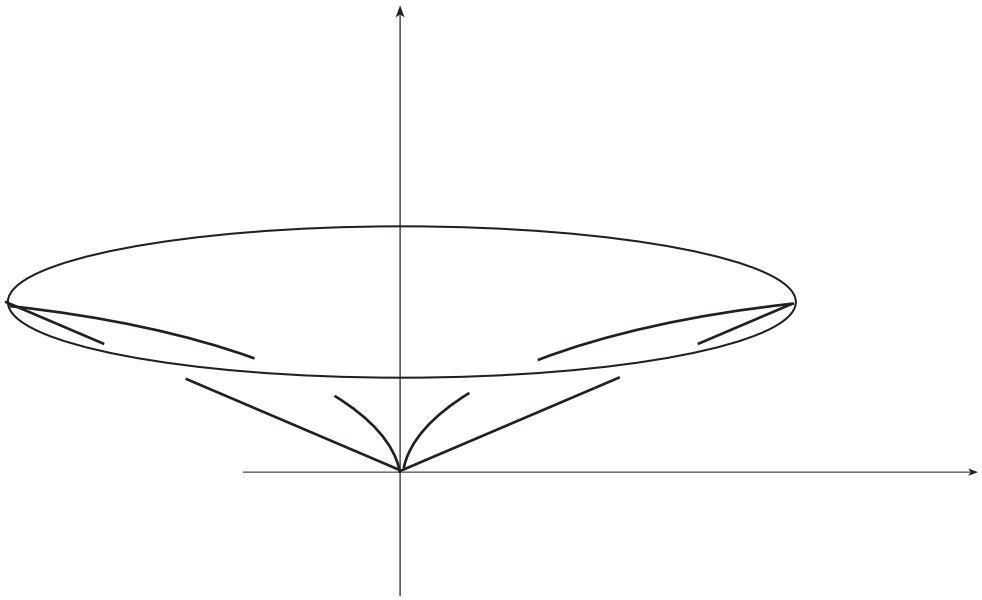


$$\begin{aligned} A &= \int_{-1}^{2} (9 - x^2) - (x + 1) dx \\ &= \int_{-1}^{2} -x^2 - x + 8 dx \\ &= -\frac{x^3}{3} - \frac{x^2}{2} + 8x \Big|_{-1}^{2} \end{aligned}$$

2. (20 points)
Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk, washer, or cylinder.

$$y^2 = x, \qquad x = 2y, \qquad \text{about the } y\text{-axis.}$$

Solution :



$$\int_0^2 \pi(2y)^2 - \pi(y^2)^2)dy = \int_0^2 4\pi y^2 - \pi y^4 dy$$
$$= \frac{4\pi}{3}y^3 - \frac{\pi}{5}y^5\Big|_0^2.$$

3. (20 points)
A 500 ft cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine-shaft 500 ft deep. Find the work done.

Solution : We need only determine the weight of the rope and coal when the coal is at a height of x, where x ranges from 0 to 500 and then integrate the result. If the coal is x units off the floor, then the rope and coal combination weighs $2(500-x)+800$. Therefore, the total work done is

$$W = \int_0^{500} 2(500-x)+800dx = 1800x - x^2\Big|_0^{500} = 650000\text{ft-lb}$$

4. (20 points)
Find the average value of the function $f(t) = t\sqrt{1+t^2}$ on the interval $[0,5]$.

Solution :

$$\begin{aligned} f_{ave} = \frac{1}{5}\int_0^5 t\sqrt{1+t^2}dt &= \frac{1}{5}\int_1^{26}\frac{1}{2}u^{1/2}du \\ &= \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{26} \\ &= \frac{26^{3/2}}{3} - \frac{1}{3}. \end{aligned}$$

5. (10 points)
Suppose g is the inverse function of f and $f(4) = 5$, $f'(4) = 2/3$. Find $g'(5)$.

Solution : Here we make use of the equation

$$g'(a) = \frac{1}{f'(g(a))}$$

with $a = 5$. Indeed, using the fact that $f(4) = 5$ implies $g(5) = 4$ we have

$$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(4)} = \frac{1}{2/3} = \frac{3}{2}.$$

6. (10 points)
Find an equation of the tangent line to the curve $y = e^{2x}\cos(\pi x)$ at the point $(0, 1)$.

Solution : The equation of such a line will be $\ell(x) = y'(0)(x-0)+1$. We only really need to compute y'. By the product rule and the chain rule $y'(x) = 2e^{2x}\cos(\pi x) - e^{2x}\pi\sin(\pi x)$. So, $y'(0) = 2$ and the line is

$$\ell(t) = 2x + 1$$