MATH 321 Section 01
Homework 1

Below is a list of problems that I will collect Friday January 26. You should write up solutions carefully and neatly and ***staple your work***. All problems from the homework are fair game on the exams! You are encouraged to work in groups, but you must write up your own solutions. I will be available during office hours for help.

(1) Either read the directions on your calculator or go online and find some freeware in an effort to find a computational tool to aid you in matrix arithmetic.

(2) Describe geometrically the set of solutions to the following systems of equations (treat the a_{ij}'s and b_i's as constants). Is the solution set a point, intersection of lines, etc...

(a) $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

(b) $a_{11}x_1 = b_1$

(c) $\left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \\ a_{31}x_1 + a_{32}x_2 = b_3 \end{array} \right\}$

(d) $\left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 = b_3 \end{array} \right\}$

(3) Find the augmented matrix for each of the following systems of linear equations.

(a) $\left\{ \begin{array}{l} x_1 - 2x_2 = 1 \\ -2x_1 + x_2 = 4 \end{array} \right\}$

(b) $\left\{ \begin{array}{l} x_1 - 2x_2 + x_4 = 0 \\ -2x_2 + 3x_3 + 5x_1 = 1 \\ x_5 = 2 \end{array} \right\}$

(c) $\left\{ \begin{array}{l} x_1 = 1 \\ x_2 = 2 \\ x_3 = 1 \end{array} \right\}$

(4) Find a system of linear equations corresponding to the augmented matrix.

(a) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 2 & 1 & -3 & 5 \\ 1 & 2 & 4 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

(5) Solve each of the following by reducing a matrix to reduced row-echelon form.

(a) $\left\{ \begin{array}{l} 2x_1 - 3x_2 = -2 \\ 2x_1 + x_2 = 1 \\ 3x_1 + 2x_2 = 1 \end{array} \right\}$

(b) $\left\{ \begin{array}{l} 3x_1 + 2x_2 - x_3 = -15 \\ 5x_1 + 3x_2 + 2x_3 = 0 \\ 3x_1 + x_2 + 3x_3 = 11 \\ -6x_1 - 4x_2 + 2x_3 = 30 \end{array} \right\}$

(c) $\left\{ \begin{array}{l} 4x_1 - 8x_2 = 12 \\ 3x_1 - 6x_2 = 9 \\ -2x_1 + 2x_2 = 1 \end{array} \right\}$

(d) $\left\{ \begin{array}{l} 10y - 4z + w = 1 \\ x + 4y - z + w = 2 \\ 3x + 2y + z + 2w = 5 \\ -2x - 8y + 2z - 2w = -4 \\ x - 6y + 3z = 1 \end{array} \right\}$

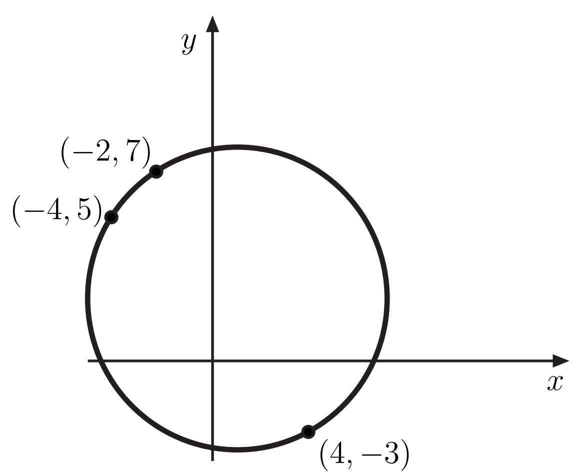
(6) Reduce the following matrix to reduced row-echelon form without introducing any fractions.

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & 7 \\ 3 & 4 & 5 \end{bmatrix}$$

(7) Recall from plane geometry that three points, not all lying on a straight line, determine a unique circle. It is shown in analytic geometry that a circle in the xy-plane has an equation of the form

$$ax^2 + ay^2 + bx + cy + d = 0.$$

Find an equation of the circle shown in the figure.



(8) Show that if $ad - bc \neq 0$, then the reduced row-echelon form of

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \text{is} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(9) Consider the following matrices:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \end{bmatrix} \qquad D = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Either compute the indicated arithmetic or explain why it is not possible.

(a) $A + B$
(b) $A + C$
(c) $3A$
(d) AB
(e) BA
(f) AC
(g) CA
(h) AD
(i) DA
(j) AA
(k) AAA

(10) Let A be any $m \times n$ matrix. Suppose that we tried to define the reciprocal of A by replacing each entry of A with its reciprocal. That is, if $A = (a_{ij})_{m\times n}$ then

$$\frac{1}{A} = \left(\frac{1}{a_{ij}}\right)_{m\times n}.$$

Is this always possible? Can you compute the reciprocal matrix for A, B, C, and D from the previous exercise without bursting into flames?

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