MATH 321 Section 01
Homework 2

Below is a list of problems that I will collect Friday February 2. You should write up solutions carefully and neatly and ***staple your work***. All problems from the homework are fair game on the exams! You are encouraged to work in groups, but you must write up your own solutions. I will be available during office hours for help.

(1) Show that if A is a square matrix and r and s are integers, then
$$A^rA^s = A^{r+s} \text{ and } (A^r)^s = A^{rs}.$$

(2) Show that if A is an invertible matrix, then
  (a) A^{-1} is invertible and $\left(A^{-1}\right)^{-1} = A$.
  (b) A^n is invertible and $(A^n)^{-1} = (A^{-1})^n$ for $n = 0, 1, 2, \ldots$
  (c) For any nonzero scalar k, the matrix kA is invertible and $(kA)^{-1} = {}^1\!/\!_k A^{-1}$.

(3) Show that if A is an invertible matrix, then A^T is also invertible and
$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T.$$

(4) Prove that the 2×2 matrix
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
is invertible if $ad - bc \neq 0$, in which case the inverse is given by the formula
$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}.$$

(5) Let A and B be square matrices of the same size. Is $(AB)^2 = A^2B^2$ a valid matrix identity? Justify your answer.

(6) Suppose that $d_1d_2d_3 \neq 0$ and consider the diagonal matrix
$$D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}.$$
Compute D^{-1}.

(7) Consider the matrices
$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 8 & 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 1 & 5 \\ 2 & -7 & -1 \\ 3 & 4 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & -7 & -1 \\ 2 & -7 & 3 \end{bmatrix}$$
Find elementary matrices E_1, E_2, E_3, and E_4 such that
  (a) $E_1A = B$
  (b) $E_2B = A$
  (c) $E_3A = C$
  (d) $E_4C = A$

(8) Find the inverse of the given matrix if the matrix is invertible and check your answer by multiplication.

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$

(9) Consider the matrix

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}.$$

Solve the equation $A\vec{x} = \vec{b}$ when $\vec{b}$ is the matrix

(a) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

(10) Show that

$$A = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix}$$

is not invertible for any values of the entries.

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